University of Bahrain

College of Information Technology
Department of Computer Science

ITCS253 Discrete Structures II
Second Semester 2014/2015

Exam #2 – 75 Minutes

STUDENT NAME	
STUDENT#	
SECTION	
SERIAL	

This exam contains **5 pages** (including this cover page) and **5 questions**. Check to see if any pages are missing. Enter all requested information on the top of this page.

You are allowed to use Calculators.

You *are not allowed* to use books, notes, or mobiles

Question	Points	Score
1	7	5
2	7	2.5
3	7	7
4	7	6
5	7	4
Total:	35	24.5

Instructor: Dr. Ali Alsaffar Sections# 1 & 2

Answer all questions

(1) Answer the following questions.

(a) **[1 point]** Let $a_n - 2\sqrt{n}a_{n-1} = 3$. Is the relation linear? Why?

1

Yes, because all term ~~dsyp~~ a_n are appear is itself only and doesn't multiply with another a_n terms.

(b) **[2 points]** Suppose a recurrence relation a_n has the characteristic equation $(x-3)(x-7) = 0$. What is a_n.

$x = 3 \quad , \quad x = 7$

1

$\therefore a_n = C_1 \cdot 3^n + C_2 \cdot 7^n$ $\quad\quad a_n = 3a_{n-1} + 7^n$

(c) **[2 points]** When we say a graph G has an Euler cycle?

1

We say the graph G has an euler cycle when all ~~ver~~ vertices of it has even degree.

and connected

(d) **[2 points]** Name two invariants of isomorphism.

1 - has n vertices ~~and~~ (that mean have same number of vertices)

2 - Is connected

2

(2) Answer all of the following questions.

(a) **[2 points]** A full *6-ary* tree with 19 vertices. Find the number of internal vertices i and the number of leaves l.

Number of internal vertices :

2

$n = mi + 1 \Rightarrow i = \frac{n-1}{m} = \frac{19-1}{6} = \frac{18}{6} = 3$

~~$\sum_{i=1}^{n} \deg(v_i)$~~

Number of leaves

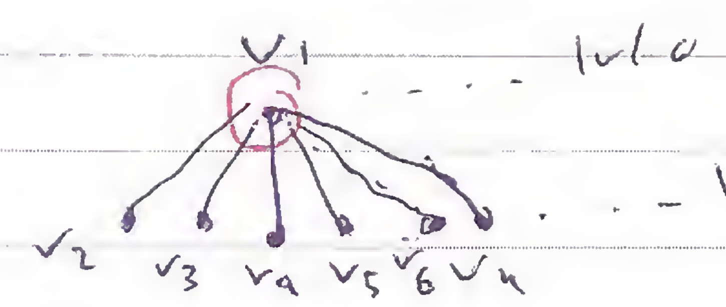
$n = l + i \Rightarrow l = n - i = 19 - 3 = 16$

(b) **[2 points]** Use the degree equation to find the number of vertices n of a tree if all of its vertices are of degree 1 (leaves). Draw the tree.

0

~~$\sum_{i=1}^{n} \deg(v_i) = 2(n-1)$~~ Graph of tree:

because the ~~trees is~~ all vertices of tree is deg (1) (leaves) except the internal vertices ~~there for~~ therefore ~~we~~ we know the height is 2 and the graph is full because there is one internal vertices and also it's complete so number of vertices



because all leaves in same level ⟹

(c) **[3 points]** Suppose a graph G is connected with n vertices and all vertices are of degree 2. Show that G is not a tree. **Hint:** Use proof by contradiction by assuming that G is a tree.

0.5

Assume G is tree and have 3 why vertices one of them of deg 2 and the remaning of degree 1. And assume ~~why~~ we put edges between those vertices of (leaves) degree one then then graph will contain cycle and tree doesn't have cycle therefore the graph will be not tree (contradiction)

(3) [7 points] Use Homogeneous technique to solve the following recurrence relation.

$$a_0 = 1, \text{ and } a_n = 5a_{n-1} + 4, \ n \geq 1$$

The charteristic equation:

$(x-5)(x-1) = 0$

The roots:

$r_1 = 5$, $r_2 = 1$

$\therefore a_n = c_1 \cdot 5^n + c_2 \cdot 1^n$

7

The initial value:

$a_1 = 5a_0 + 4 = 5 \times 1 + 4 = 9$

The boundry condition:

$a_0 = 1 \Rightarrow c_1 + c_2 = 1 \dots ①$

$a_1 = 9 \Rightarrow 5c_1 + c_2 = 9 \dots ②$

~~by multiplay ① by~~

by subtruct ① from ②:

$c_1 + c_2 = 1$

$5c_1 + c_2 = 9$

$-4c_1 + 0 = -8$

$-4c_1 = -8 \Rightarrow \boxed{c_1 = \frac{-8}{-4} = 2}$

$\therefore c_1 + c_2 = 1 \Rightarrow 2 + c_2 = 1$

$\Rightarrow \boxed{c_2 = 1 - 2 = -1}$

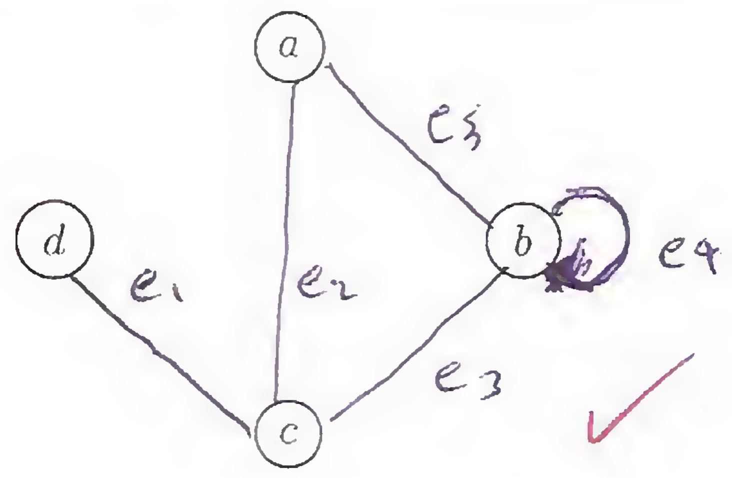
$\therefore a_n = 2 \cdot 5^n - 1^n$

(4) Suppose M is the adjacency matrix for an *undirected* graph G.

$$M = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

(a) [**2 points**] Draw the graph using the below vertex ordering.

2



Undirected Graph

(b) [**2 points**] Find the incident matrix of the graph.

2

$$G = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{ccccc} e_1 & e_2 & e_3 & e_4 & e_5 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

(c) [**1 point**] Is the graph a Bipartite graph? Why?

1

No, because verrex b adjecency itsele

loop

(d) [**1 point**] Which edges you can remove from G to form a spanning tree.

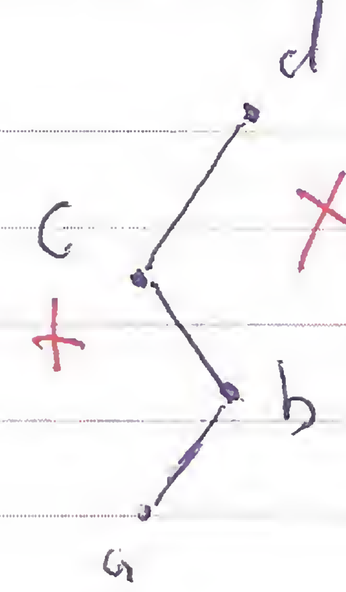
1

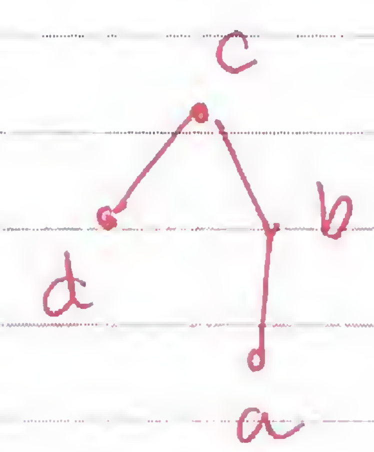
~~edges are {e_2, e_4}~~

edges are e_2 and e_4

(e) [**1 point**] Draw the spanning tree found in (d) as a rooted tree with c as the root.

0





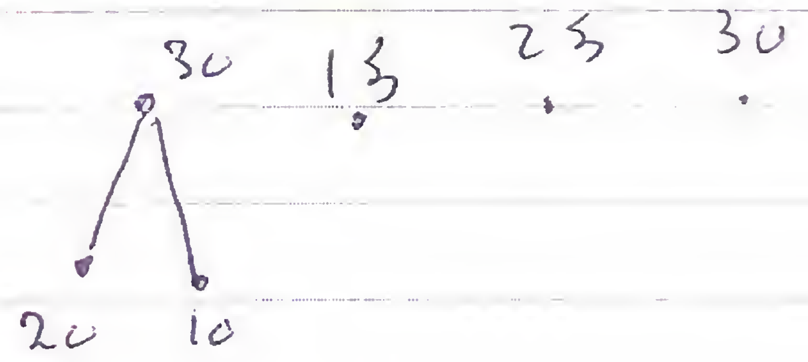
(5) (a) **[5 points]** Construct an optimal Huffman code using the table below.

Character	a	b	c	d	e
Frequency	20	10	15	25	30

1.5

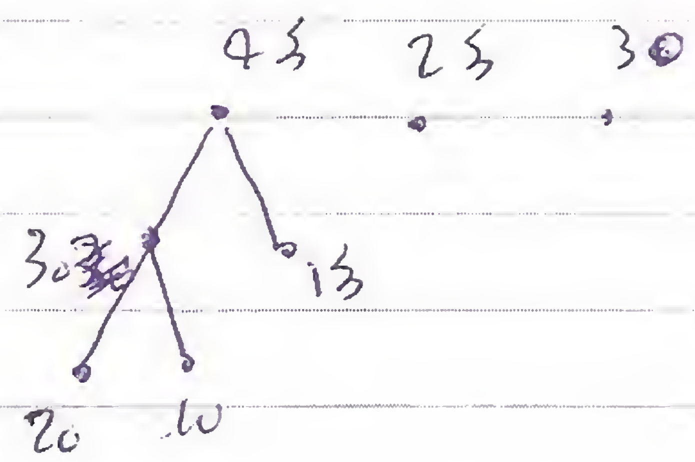
20 10 15 25 30 ← sorted ?

by sum first two:

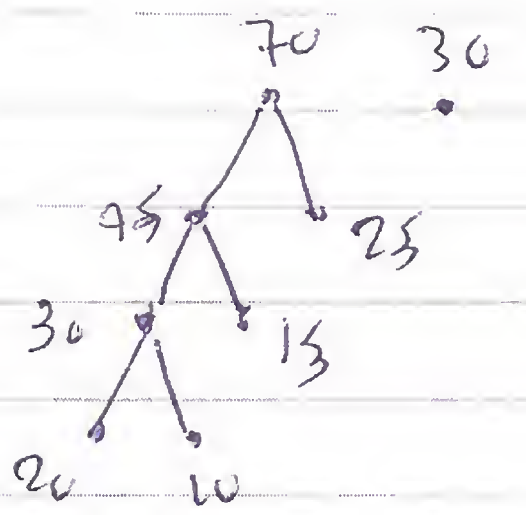


by sum first two:

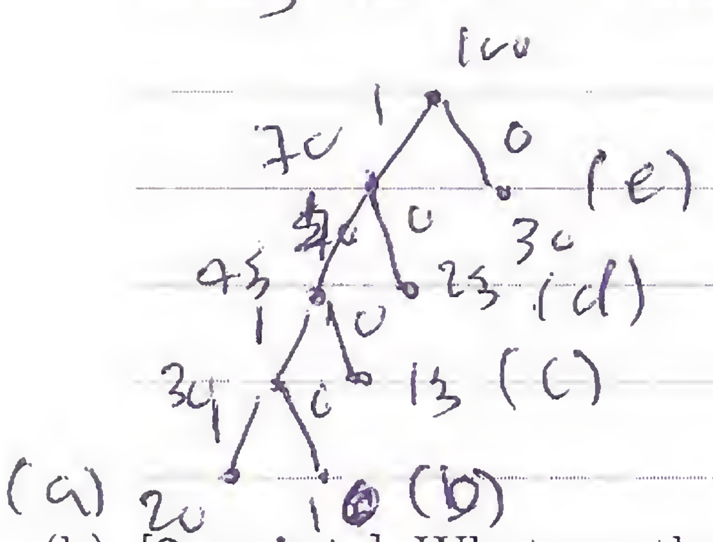
3.5



by sum first two:



by sum last two vertices



(b) **[2 points]** What are the bit codes obtained in part (a) for a, b, c, d, and e, respectively.

0.5

a	b	c	d	e
1010101	10101010	101010	1010	~~10~~ 10